Name of the Lab Course (Basic Structural Analysis Lab)

Instructors: Wassim Elias, M2R. Civil Eng.
              Youssef Kassouf, Civil Eng.

Lecture time: Tuesday 15:00-17:00 with Eng. Youssef Kassouf
Lecture time: Tuesday 17:00-19:00 with Eng. Wassim Elias
Room: Centre Informatique CIVIL
Number of hours per week: 4h
Total of hours per semester (12 weeks): 48h

Description
This laboratory gives the basic understanding of structural member under the action of loads. How structure's behavior will be changed and how suddenly it becomes unstable (failure) will be experienced by student. The main purpose of any structure is to support the loads coming on it by properly transferring them to the supports. The computation of internal and external forces and displacements produced by the application of loads to make the structure safe. Detailed analysis is to be performed to determine the bending moments, shear force, axial forces etc. at the required section. Basic structural analysis contains; linear elastic analysis of structural systems and members like, beam, column, slab, frame, footing etc.

List of Experiments:
1. Tensile Test Experiment.
2. Three Point Bending Experiment.
3. Buckling of a Column Experiment.
4. Shear Test Experiment.
5. Trusses Experiment.

Detailed schedule
The class is divided into 8 groups as follows:

<table>
<thead>
<tr>
<th>Week</th>
<th>Topic covered</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Group 1-2: Tensile Test Experiment + Three Point Bending Experiment</td>
</tr>
<tr>
<td>2</td>
<td>Group 3-4: Tensile Test Experiment + Three Point Bending Experiment</td>
</tr>
<tr>
<td>3</td>
<td>Group 5-6: Tensile Test Experiment + Three Point Bending Experiment</td>
</tr>
<tr>
<td>4</td>
<td>Group 7-8: Tensile Test Experiment + Three Point Bending Experiment</td>
</tr>
<tr>
<td></td>
<td>Group 1-2: Buckling of a Column Experiment + Shear Test Experiment</td>
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<tr>
<td>6</td>
<td>Group 3-4: Buckling of a Column Experiment + Shear Test Experiment</td>
</tr>
<tr>
<td>7</td>
<td>Group 5-6: Buckling of a Column Experiment + Shear Test Experiment</td>
</tr>
<tr>
<td>8</td>
<td>Group 7-8: Buckling of a Column Experiment + Shear Test Experiment</td>
</tr>
<tr>
<td>9</td>
<td>Group 1-2: Trusses Experiment</td>
</tr>
<tr>
<td>10</td>
<td>Group 3-4: Trusses Experiment</td>
</tr>
<tr>
<td>11</td>
<td>Group 5-6: Trusses Experiment</td>
</tr>
<tr>
<td>12</td>
<td>Group 7-8: Trusses Experiment</td>
</tr>
</tbody>
</table>

**Evaluation**

Participation and Assiduity : 25%
Lab Experiment Assignment : 75%

**Textbook**


**Supplemental materials**

Course of the Lab, calculators, graph paper, computers, Microsoft Excel…

**Additional course policies and requirements**

- The participation in Lab courses is mandatory.
- The Lab schedules must be respected.
- All absences must be justified and an extra-session will be organized.
- The use of mobile phones is strictly prohibited during the Lab courses.
- It is strictly forbidden to smoke in the halls of the Lab.
- Each Lab will be graded on 100 and the passing grade is 60.
Lab 1 – Tensile Test Experiment

Objective of the Lab

1. Measurement:
   - Position ________ (m).
   - Tensile Force ________ (KN).

2. Construction
   - Simple Bar (with a defined material: i.e.: Steel, Aluminum, Brass …).

The basic idea of a tensile test is to place a sample of a material between two fixtures called “grips” which clamp the material. The material has known dimensions, like length and cross-sectional area. We then begin to apply weight to the material gripped at one end while the other end is fixed. We keep increasing the weight (often called the load or force) while at the same time measuring the change in length of the sample.
3. Observation:
We run the test and we get from the program the table containing the force versus position.

**Theoretical study**

For each material has its own modulus of elasticity E, which can be conducted from $\sigma = \varepsilon \cdot E$.

- $\sigma =$ stress MPA=F/$S_0$ in N/mm$^2$ or MPa.
- F= Force of the weights applied on the bar in N.
- $S_0 =$ Area of the bar diameter in mm$^2$. (Diameter is equal to 3.3 mm as the standard section geometry in PASCO).
- $\varepsilon =$ Strain = $\Delta L/L_0$
- $\Delta L=$ L-L$_0$
- L= Bar length after displacement in mm.
- L$_0$= Initial Bar length in mm (90 mm as the standard section geometry in PASCO).

**Experimental Study**

The result of this test is a graph of load (amount of weight) versus displacement (amount it stretched). Since the amount of weight needed to stretch the material depends on the size of the material (and of course the properties of the material), comparison between materials can be very challenging. The ability to make a proper comparison can be very important to someone designing for structural applications where the material must withstand certain forces.

**Observation Table.**

<table>
<thead>
<tr>
<th>Position $\Delta L$ (mm)</th>
<th>Tensile Force F (N)</th>
<th>Stress $\sigma$ (Mpa)</th>
<th>Strain $\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L$_1$</td>
<td>F$_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L$_2$</td>
<td>F$_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L$_3$</td>
<td>F$_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
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<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L$_n$</td>
<td>F$_n$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
After we get the table we calculate the stress and strain, then we draw the stress-strain graph using excel to find the mechanical properties of the material subjected to the tensile test:

- Modulus of elasticity (Module de young) $E$ in Gpa.
- Limit of elasticity $R_e$ in Mpa.
- Conventional limit of elasticity $R_{e0.2}$ in Mpa.
- Traction stress $R_m$ in Mpa.
  \[ \varepsilon_r = \frac{L_f - L_0}{L_f} \]
- Elongation $\varepsilon_r$
- $L_f$ is the bar length after rupture in mm (90 mm as the standard section geometry in PASCO).

- Shear modulus $G$ in Gpa if the poisson coefficient is given where:
  \[ G = \frac{E}{2(1+\nu)} \]

- Strain Energy $W$ in J/m$^3$
  \[ W = \int_0^\varepsilon \sigma \varepsilon = \frac{1}{2} \rho \varepsilon = \frac{\sigma^2}{2E} \]
Lab 2 – Three Point Bending Experiment

Objective of the Lab

1. Measurement:
   - Force ________ (N).
   - Position ________ (mm).

2. Construction
   - Simple Bar (with a defined material: i.e.: Steel, Aluminum, plastic …).

The test method for conducting the test usually involves a specified test fixture on a universal testing machine. Details of the test preparation, conditioning, and conduct affect the test results. The sample is placed on two supporting pins a set distance apart and a third loading pin is lowered from above at a constant rate until sample failure.

3. Observation:

We run the test and we get from the program the table containing the force versus position then we draw the graph in excel and we get the flexural modulus of elasticity \( E_f \).
Theoretical study

- Calculation of the flexural stress $\sigma_f = \frac{3FL}{2bd^2}$ for a rectangular cross section (Width b and Height d).
- Calculation of the flexural stress $\sigma_f = \frac{FL}{\pi R^4}$ for a circular cross section (Radius R).

- Calculation of the flexural strain $\epsilon_f$ for a rectangular cross section: $\epsilon_f = \frac{6Dd}{FL}$
- Calculation of flexural modulus $E_f$ for a rectangular cross section: $E_f = \frac{L^2m}{4bd^3}$
- Calculation of flexural modulus $E_f$ for a circular cross section: $E_f = \frac{mL^3}{(12 \pi R^4)}$
- Calculation of the flexural strain $\epsilon_f$ for a circular cross section: $\epsilon_f = \frac{12F.R}{(m.L^2)} = \frac{12D.R}{L^2}$

In these formulas the following parameters are used:
- $\sigma_f$ = Stress in outer fibres at midpoint, (Mpa)
- $\epsilon_f$ = Strain in the outer surface, (mm/mm)
- $E_f$ = flexural Modulus of elasticity,(Mpa)
- $F$ = load at a given point on the load deflection curve (Charge), (N)
- $L$ = Support span, (mm) (the sample standard length is equal to 64.5 mm)
- $b$ = Width of test beam, (mm)
- $d$ = Depth or thickness of tested beam, (mm)
- $D$ = maximum deflection of the centre of the beam (Flèche), (mm) = $\frac{FL^3}{(48EI)}$
- $m$ = The gradient (i.e., slope) of the initial straight-line portion of the load deflection curve,( F/D), (N/mm)
- \( R \) = The radius of the beam, (mm) (the sample standard radius is equal to 3.33/2 mm)

**Experimental Study**

The three point bending flexural test provides values for the modulus of elasticity in bending \( E_f \), flexural stress \( \sigma_f \), flexural strain \( \varepsilon_f \) and the flexural stress-strain response of the material. The main advantage of a three point flexural test is the ease of the specimen preparation and testing. However, this method has also some disadvantages: the results of the testing method are sensitive to specimen and loading geometry and strain rate.

From the test we get the table containing force \( F \) versus position \( D \) then we draw the graph on the excel and we find the maximum deflection of the centre of the beam \( D \) and its corresponding force \( F \) that caused the rupture in order to find the flexural Modulus of elasticity \( E_f = \frac{4Fm}{kB^3} \) in GPa for a rectangular cross section beam, and \( E_f = \frac{mL^4}{12 \pi R^4} \) in GPa for a circular cross section beam.

If the specimen does not break, the test is continued as far as possible and the stress at 3.5% (conventional deflection) is reported.

**Observation Table.**

<table>
<thead>
<tr>
<th>Load/Deflection F (N)</th>
<th>D (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>D1</td>
</tr>
<tr>
<td>F2</td>
<td>D2</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>Fn</td>
<td>Dn</td>
</tr>
</tbody>
</table>

**Output Results:** Flexural stress at yield, flexural strain at yield, flexural stress at break, flexural strain at break, flexural stress at 3.5% deflection, flexural modulus, Stress/Strain curves and raw data can be provided.
Lab 3 – Buckling of a Column Experiment

Objective of the Lab

1. Measurement:
   - Force ________ (N).
   - Position ________ (mm).
2. Construction
   - Simple Bar (with a defined material: i.e.: plastic (as the standard section geometry in PASCO)).

The test method for conducting the test usually involves a specified test fixture on a universal testing machine. The test device mainly consists of a basic frame, the guide columns and the load cross bar. The basic frame contains the bottom mounting for the rod specimen, consisting of a force-measuring device for measuring the testing force. The height of the load cross bar can be adjusted along the guide columns and it can be clamped in position. This allows rod specimens with different buckling lengths to be examined.

The objective of this experiment is:
   a. To determine critical buckling loads for columns with support.
   b. To examine the Euler theory of buckling.
   c. To investigate the influence of different material parameters.
All relevant buckling problems can be demonstrated with the WP 120 test (as the standard section geometry in PASCO). Buckling, as opposed to simple strength problems such as drawing, pressure, bending and shearing, is primarily a stability problem. Buckling problem number among the best-known technical examples in stability theory. Buckling plays an important role in almost every field of technology. Examples of this are:

- Columns and supports in construction and steel engineering
- Stop rods for valve actuation and connecting rods in motor construction
- Piston rods for hydraulic cylinders and
- Lifting spindles in lifting gear

3. Observation:

We run the test and we get from the program the table containing the force versus position then we draw the graph in excel and we get the critical limit load $F_{\text{crit}}$ and the effective length from the test.

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>End condition</th>
<th>Euler’s Buckling load</th>
<th>Effective Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Theoretical</td>
<td>Observed</td>
</tr>
<tr>
<td>1</td>
<td>Both ends fixed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>One end fixed and other pinned</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Both ends pinned</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>One end fixed and other free</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Theoretical study**

a. Applying the Buckling Theory If a rod is subjected to longitudinal forces, as implied in the sketch, it can fail in two ways. On the one hand, it can be plasticized and flattened if its admissible compressive strain is exceeded (see Figure below). On the other hand, it is possible that it will suddenly shift to one side and buckle before attaining the admissible compressive strain. This effect is called buckling. The shape of the rod is the factor determines which of the two cases of failure will occur. A slender, thin rod is more likely to buckle than a thick, stout rod
b. Euler Formula
Buckling occurs suddenly and without warning when a certain limit load is attained. It is therefore an extremely dangerous type of failure, which must be avoided by all means. As, soon as a rod begins to buckle, it will become deformed to the point of total destruction. This is typical unstable behavior. Buckling is a stability problem. The critical limit load \( F_{\text{crit}} \), above which buckling can occur is dependent on both the slenderness of the rod, i.e. influence of length and diameter, and the material used. In order to define slenderness the slenderness ratio \( \lambda \) will be introduced here.

\[
\lambda = \frac{l_k}{i}
\]

In this case \( l_k \) is the characteristic length of the rod. It takes both the actual length of the rod and the mounting conditions into consideration.

For example, clamping the ends of the odds causes rigidity. The buckling length decisive for slenderness is shorter than the actual length of the rod. Altogether, a differentiation is made between four types of mountings, each having a different buckling length.

The influence of diameter in the slenderness ratio is expressed by the inertia radius \( i \). It is calculated using the minimum geometrical moment of inertia \( I_y \) and the cross-sectional area \( A \).

\[
i = \sqrt{\frac{I_y}{A}}
\]
The influence of material is taken into consideration by the longitudinal rigidity of the rod $E A$. Here, $E$ is the modulus of elasticity of the respective material and $A$ is the cross-sectional area. The influence of various factors on the critical load is summarized in the so-called “Euler formula”:

$$F_{\text{crit}} = \pi^2 \frac{EA}{\lambda^2}$$

or expressed in a different form:

$$F_{\text{crit}} = \pi^2 \frac{El_y}{l^2}$$

c. **Influencing Factors**
Below the influence of various characteristic values such as the $E$ modulus, geometric moment of inertia, length and the type of mounting on buckling behavior will be examined using the Euler formula.

d. **Tensions in Buckling Rod**
In order to determine whether a rod has failed due to exceeding the admissible compressive strain or by buckling, the normal compressive strain in the rod, which is part of the critical load, must be calculated.

$$\sigma_k = \frac{F_k}{A} = \pi^2 \frac{E}{\lambda^2}$$

If this normal compressive strain is lower than the admissible compressive strain, the rod will fail due to buckling. If the admissible compressive strain is used as the normal compressive strain, the critical slenderness ratio $\lambda_{\text{crit}}$ at which buckling occurs can be calculated.

$$\lambda_{\text{crit}} = \sqrt{\pi^2 \frac{E}{\sigma_p}}$$

**Ex:** For constructive steel St37 with $\sigma_p = 192$ N/mm the $\lambda_{\text{crit}} = 104$. Above $l_{\text{crit}}$ buckling according to Euler can be expected. The buckling strain curve can be seen in the diagram below.
Experimental Study

In this test the operation of the WP 120 buckling test device and how to conduct a buckling test is demonstrated.

A rod with articulated mounting at both ends cording to Euler case 3 (Fixed supports) is slowly subjected to an axial force. Above a certain load it will buckle laterally. In this case the buckling (deformation) of the rod specimen will be measured in the middle of the rod and recorded in a table along with the accompanying force. A force/deformation graph will be developed using these measured values. The results of the test should be compared with the buckling theory values.
Lab 4 – Shear Test Experiment

Objective of the Lab

1. Measurement
   - Force ________ (N).
   - Position ________ (mm).

2. Construction
   Place a circular bar (Brass, aluminium or steel) in the shear accessory extended 6 mm from each of the two outer edges.

Shear Accessory and Sample Bars

3. Observation
Theoretical study

\[ s = \left( \frac{\delta M}{\delta x} \right) \frac{A \bar{y}}{z I} = F \cdot \frac{A\bar{y}}{z I} \]  

(3)

Note: \( F = \frac{dM}{dx} \). See "Shearing Force and Bending Moments".

It should also be noted that \( Z \) is the actual width of the section at the position where \( s \) is being calculated and that \( I \) is the Total Moment of Inertia about the neutral axis. In some applications it is advantageous to calculate \( A \bar{y} \) as several parts.

Rectangular Sections.

For a rectangular section at any distance \( y \) from the Neutral Axis:

\[ A = b \left( \frac{d}{2} - y \right) \]

\[ bary = \frac{1}{2} \left( \frac{d}{2} + y \right) \text{ and } z = b \]

Substituting in equation (3) and putting \( I = \frac{b \, d^3}{12} \)

\[ s = \frac{F(d/2 - y)(d/2 + y)}{b \times \left( \frac{b \, d^3}{12} \right) \times 2} \]

\[ \therefore s = \left( \frac{6 \, F}{b \, d^3} \right) \left( \frac{d^2}{4} - y^2 \right) \]

This shows that there is a parabolic variation of Shear Stress with \( y \).

The maximum Shear Force occurs at the Neutral Axis and is given by:

\[ \bar{s} = \frac{3 \, F}{2 \, b \, d} \]
Solid Circular Sections.

Let \( \sigma \) be the Shear Force across a chord parallel to \( XX \) defined by the angle \( \theta \)

\[
\sigma = F \times \frac{A y}{z I} = \frac{F}{2R \cos \theta} \int \left((2x \ dy) \ y \right) dA = \frac{4}{R^5 \cos \theta} \int_{R \sin \theta}^R \sqrt{(R^2 - y^2)} y \ dy
\]

\[
= \frac{4}{\pi R^5 \cos \theta} \left[ \frac{1}{3} (R^2 - y^2)^{\frac{3}{2}} \right]_{R \sin \theta}^R
\]

\[
= \frac{4}{\pi R^5 \cos \theta} \times \frac{1}{3} (1 - \sin^2 \theta)^{\frac{3}{2}} = \frac{4 F \cos^2 \theta}{3 \pi R^2}
\]

At the Neutral Axis the Shear Stress \( \sigma \) is maximum and equals \( \frac{4 F}{3 \pi R^2} \)

Experimental Study

Apply a force \( F \) in (N) and record the stress \( \tau = F/A \) in (MPa) versus the position \( D \) in (mm) and observe the Stress-Position curve in order to find the maximum shear stress at the rupture point. With \( A \) is the cross section area of the sample (rectangular or circular).
Lab 5 – Trusses Experiment

Objective of the Lab

A truss is a structure comprising one or more triangular units constructed with straight members whose ends are connected at joints referred to as nodes. External forces and reactions to those forces are considered to act only at the nodes and result in forces in the members which are either tensile or compressive forces. Moments (torsional forces) are explicitly excluded because, and only because, all the joints in a truss are treated as revolute. A planar truss is one where all the members and nodes lie within a two dimensional plane (2D), while a space truss has members and nodes extending into three dimensions (3D). A truss is composed of triangles because of the structural stability of that shape and design. A triangle is the simplest geometric figure that will not change shape when the lengths of the sides are fixed. In comparison, both the angles and the lengths of a four-sided figure must be fixed for it to retain its shape.

This introductory Structures System set in PASCO includes everything needed to quickly build a variety of trusses. A variety of I-beams give the flexibility to design and create many different structures. As with all Structures Systems sets, Load Cells (sold separately) can be placed anywhere in the structure, allowing you to measure tension and compression forces at that point.

1. Measurement
   - Force ________ (N).
   - Position _________ (mm).

2. Construction
Construct the truss using PASCO plastic bar elements numbered from 1 to 5 depending on their lengths. Easy as 1-2-3 to construct as follows:

- #1 (5.5 cm long) I-beam (8)
- #2 (8 cm long) I-beam (8)
- #3 (11.5 cm long) I-beam (18)
- #4 (17 cm long) I-beam (18)
- #5 (24 cm long) I-beam (8)

- Fit the I-beams into the connectors
- Secure the beams with thumb screws
- Thumb screws are also slotted for use with a screwdriver

and install some load cells on specific bar elements of the truss in subject in order to measure the bar axial forces $N$ using PASCO (note that the load cells are connected to a load cell amplifier Xplorer GLX datalogger machine, which can then be connected to any current PASCO interface).

Some unit loads are applied using weights hanged on the middle of each specified joint of the truss.

3. Observation

The Load Cell & Amplifier Set provides you the means to accurately measure tension and compression in real-time—anywhere in your structure! You build the Load Cell right into your structure where you want it—and easily relocate it without disassembling the entire structure.

**Theoretical study**

The basic building block of a truss is a triangle. Large truss are constructed by attaching several triangles together. A new triangle can be added truss by adding two members and a joint. A truss constructed in this fashion is known as a simple truss. A truss is analyzed by using $m = 2j - 3$, 
where \( m \) is number of members, \( j \) represents the number of joints and \( 3 \) represents the external support reactions. Plane trusses lie in a single plane. In straight members forces act along the axis of the member. Compressive forces tend to shorten the member. Tensile forces tend to elongate the member. Space trusses: not contained in a single plane and/or loaded out of the structure plane. There are four main assumptions made in the analysis of truss

1. Truss members are connected together at their ends only.
2. Truss is connected together by friction less pins.
3. The truss structures is loaded only at the joints
4. The weights of the members may be neglected.

Techniques for Truss Analysis
- Method of joints: usually used to determine forces for all members of truss
- Method of sections: usually used to determine forces for specific members of truss
- Determining Zero-force members: members who do not contribute to the stability of a structure
- Determining conditions for analysis: is the system statically determinate?

The deflection of a node of a truss under a given loading is determined by:
\[
\delta = \Sigma \left( \frac{nNL}{AE} \right)
\]
Where, \( \delta \) = deflection at the node point.
\( n \) = Force in any member under a unit load applied at the point where the deflection is required.
The unit load acts when the loading on the truss have been removed and acts in the same direction in which the deflection is required.
\( N \) = Force in any member under the given loading.
\( L \) = Length of any member.
\( A \) = Cross sectional area of any member.
\( E \) = Young's modulus of elasticity of the material of the member.

Use the virtual work method to find the force in the truss members.

**Experimental Study**

Apply weights in (N) and record the force in the truss members in (N) in order to compare it with the theoretical method using Virtual Work Method.